Let D_{opt} be an optimal disc of radius r_{opt} centered at m_{opt}. Let P be a polygon in $\mathcal{P}$. Let e be an edge of P intersected by D_{opt} and l be the line supporting e. We assume w.l.o.g. that l is the x-axis. For $\delta > 0$ we define the following family of lines $\mathcal{L}_\delta = \{l_\delta^i \,|\, i \in \mathbb{Z}\}$ where $l_\delta^i = \{(x, i\delta) \in \mathbb{R}^2 \,|\, x \in \mathbb{R}\}$.

Assume m_{opt} lies between l_δ^{i-1} and l_δ^i. Since $d(m_{\text{opt}}, l) \leqslant r_{\text{opt}}$ it follows that $|i| \leqslant \lceil r_{\text{opt}}/\delta \rceil$. Let m be the vertical projection of m_{opt} onto l_δ^i. The disc D of radius $r = r_{\text{opt}} + \delta$ around m contains the disc D_{opt} and therefore touches all polygons in $\mathcal{P}$. Thus for any $B \geqslant r_{\text{opt}}$ there is a disc D of radius $r = r_{\text{opt}} + \delta$ touching all polygons in $\mathcal{P}$ that is centered at a point m that lies on one of the $O(B/\delta)$ many lines in $\mathcal{L}_\delta^B = \{l_\delta^i \,|\, i \in \mathbb{Z}, |i| \leqslant \lceil B/\delta \rceil\}$.

Suppose we know a value r_{app} with $r_{\text{opt}} \leqslant r_{\text{app}} \leqslant 2r_{\text{opt}}$. Fix an arbitrary $\varepsilon \geqslant 0$. We set $B = r_{\text{app}}$ and $\delta = \varepsilon r_{\text{app}}/2$. Then the best solution on the $O(1/\varepsilon)$ lines in $\mathcal{L}_{\varepsilon r_{\text{app}}/2}^{r_{\text{app}}}$ has a radius of at most $(1+\varepsilon) r_{\text{opt}}$. It can be computed in $O((n/\varepsilon)\text{polylog}(n))$ time.

In general, we do not know e of course. There are two ways to proceed:

First observe, that P can be an arbitrary polygon in $\mathcal{P}$. Thus, we can choose P to be the polygon with the smallest number of vertices, and try all the edges of P as candidates for e. There are $O(n/m)$ such candidate edges and the overall runtime for computing a $(1+\varepsilon)$-approximation to r_{opt} with this approach is therefore $O(n^2/(\varepsilon m)\text{polylog}(n))$.

In a second approach we *randomly* choose an edge e from the n edges of the polygons in $\mathcal{P}$ and proceed as above to compute a solution that lies on a line parallel to e in $O((n/\varepsilon)\text{polylog}(n))$ time. We call e *good* if it is intersected by D_{opt}. If e is a good edge, we get a $(1+\varepsilon)$-approximation to r_{opt} (otherwise we do not know what we get). Since each polygon has at least one good edge, the probability that e is good is at least $1/m$. If we repeat this experiment $O(m)$ times, we find a good edge with high probability. The overall runtime for computing (with high probability) a $(1+\varepsilon)$-approximation to r_{opt} with this approach is $O((mn/\varepsilon)\text{polylog}(n))$. Thus (given r_{app}) we can compute (w.h.p.) a $(1+\varepsilon)$-approximation to D_{opt} in $O((1/\varepsilon)\min(mn, n^2/m)\,\text{polylog}(n))$ time.

It remains to explain how we get a 2-approximation r_{app} to r_{opt}.

Let D_{opt} be an optimal disc of radius r_{opt} centered at m_{opt}. Let P be a polygon in $\mathcal{P}$. Let e be an edge of P intersected by D_{opt} and l be the line supporting e. Let m be the vertical projection of m_{opt} onto l. The disc D of radius $r = r_{\text{opt}} + d(m_{\text{opt}}, l)$ around m contains the disc D_{opt} and therefore touches all polygons in $\mathcal{P}$. Since $d(m_{\text{opt}}, l) \leqslant r_{\text{opt}}$ this is a 2-approximation. Since we do not know e we have to proceed as above to find it (w.h.p.). Thus we can compute (w.h.p.) a 2-approximation to D_{opt} in $O(\min(mn, n^2/m)\,\text{polylog}(n))$ time.